CSE 150A 250A
AI: Probabilistic Methods

Fall 2025

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Slides adapted from previous versions of the course (Prof. Lawrence, Prof. Alvarado, Prof Berg-Kirkpatrick)

Probability Review

Probability in AI

Probability Theory == "How knowledge affects belief" (Poole and Mackworth)

What is the probability that it is raining out?

How much do I believe that it is raining out?

Viewing probability as measuring belief (rather than frequency of events) is known as the Bayesian view of probability (as opposed to the frequentist view).

Discrete Random Variables

Discrete random variables, denoted with capital letters: e.g., X

Domain of possible values for a variable, denoted with lowercase letters: e.g., $\{x_1, x_2, x_3, \ldots, x_n\}$

Example: Weather W ; $\{w_1 = sunny, w_2 = cloudy\}$

Unconditional (prior) Probability

marginal

$$P(X = x)$$

e.g., What is the probability that the weather is sunny?

$$P(W = w_1)$$

Axioms of Probability

$$P(X = x) \ \geq 0$$

$$\sum_{i=1}^{n} P(X = x_i) = 1$$

$$P\left(X = x_i \ or \ X = x_j\right) = P(X = x_i) + P\left(X = x_j\right) \ if \ x_i \neq x_j$$

Mutually Exclusive!

Conditional Probability

given

$$P(X = x_i | Y = y_j)$$

"What is my belief that $X = x_i$ if I already know $Y = y_j$"

Sometimes, knowing Y gives you information about X, i.e., changes your belief in X. In this case X and Y are said to be *dependent*.

$$P(X = x_i | Y = y_j) \neq P(X = x_i)$$

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Course code:
KSALDG

Axioms of Conditional Probability

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Which of the following axioms hold for conditional probabilities?

A. $P(X = x_i | Y = y_j) \geq 0$

B. $\sum_i P(X = x_i | Y = y_j) = 1$

C. $\sum_j P(X = x_i | Y = y_j) = 1$

D. *A and B only*

E. *A, B and C*

$\sum_i P(X = x_i) = 1$

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Marginal Independence

$$P(X = x_i | Y = y_j) = P(X = x_i)$$

Sometimes knowing Y does not change your belief in X. In this case, X and Y are said to be *independent*.

$$P(W = w_i | Y = y_j) = P(W = w_i)$$

If W denotes the weather today, For which variable Y is the above statement most likely true?

A. Y = The weather yesterday

B. Y = The day (Mon, Tue…) of the week

C. Y = The temperature

More independence

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Consider two students Roberto and Sabrina, who both took the same test. Define the following random variables:

R = Roberto aced the test

S = Sabrina aced the test

Assume both students have similar* ability and the only deciding factor for acing the test is the ***test difficulty*.*

What is the most logical relationship between $P(R = 1)$ and $P(R = 1|S = 1)$?

A. $P(R = 1) = P(R = 1|S = 1)$

B. $P(R = 1) > P(R = 1|S = 1)$

C. $P(R = 1) < P(R = 1|S = 1)$

Conditional Independence

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What if you also know the test was easy (variable T)?

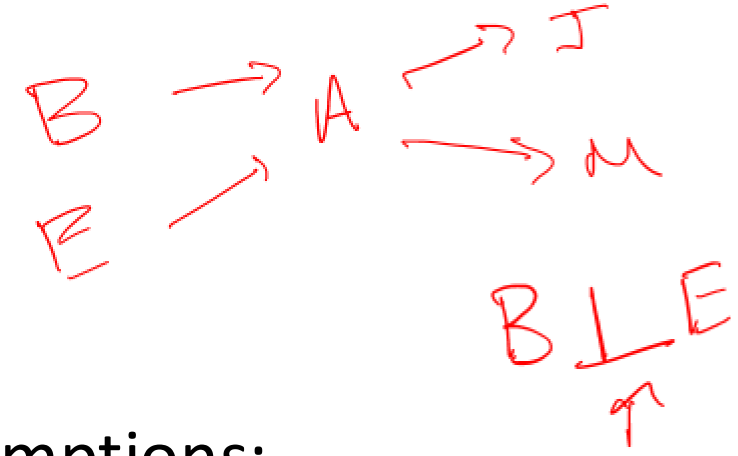
A. $P(R = 1|T = 1) = P(R = 1|T = 1, S = 1)$

B. $P(R = 1|T = 1) > P(R = 1|T = 1, S = 1)$

C. $P(R = 1|T = 1) < P(R = 1|T = 1, S = 1)$

R and S are conditionally independent given T. i.e., if you already know T, knowing S does not give you additional information about R.

More independence



Consider these random variables:

- **B** = Burglary occurs
- **E** = Earthquake occurs
- **A** = Alarm goes off
- **J** = Jamal calls
- **M** = Maya calls

Assumptions:

- Burglary and earthquake are independent. (B⊥E)
- Alarm can be triggered by either burglary or earthquake.
- Jamal and Maya call if they hear the alarm.
- Jamal and Maya do not directly influence each other.

Cumulative Evidence

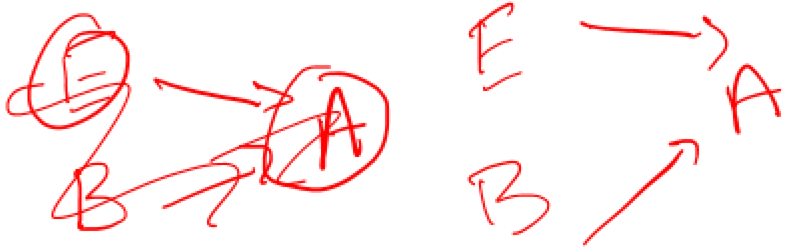
If you know there was an earthquake, what happens to your belief about the alarm going off? Does it increase, decrease or stay the same?

$$P(A = 1) \underline{\leq} P(A = 1|E = 1)$$

If you know that there was an earthquake **AND** that there was a burglary. What happens to your belief about the alarm going off?

$$P(A = 1|E = 1) \underline{\leq} P(A = 1|E = 1, B = 1)$$

Explaining Away



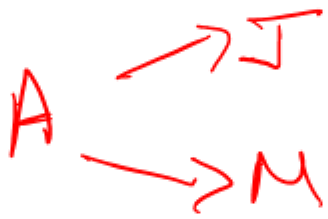
If the alarm goes off, what happens to your belief about the burglary?

$$P(B = 1) \underline{\ \leq\ } P(B = 1|A = 1)$$

Now, if the alarm goes off, and you also get to know there was an earthquake – what happens to your belief about the burglary?

$$P(B = 1|A = 1) \underline{\ \geq\ } P(B = 1|A = 1, E = 1)$$

Conditional Independence



- If you don't know the alarm status, does Jamal's call change your belief about Maya calling?

$$P(M = 1) \underline{\leq} P(M = 1|J = 1)$$

- If you know the alarm went off, does Jamal's call still change your belief about Maya calling?

$$P(M = 1|A = 1) \underline{=} P(M = 1|A = 1, J = 1)$$

Independence

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$$P(X = x_i | Y = y_j) \neq P(X = x_i) \Rightarrow$$
$$P(X = x_i | Y = y_j, Z = z_k) \neq P(X = x_i | Z = z_k)\ ?$$

If X and Y are dependent, does this imply X and Y are dependent given Z?

A. Yes

B. No

Independence

Course code: **GJLOWD**

$$P(X = x_i | Y = y_j) = P(X = x_i) \Rightarrow$$
$$P(X = x_i | Y = y_j, Z = z_k) = P(X = x_i | Z = z_k)\ ?$$

If X and Y are independent, does this imply X is independent of Y given Z?

A. Yes
B. No

Review - Independence

- Marginal Independence
  - $P(X = x_i | Y = y_j) = P(X = x_i)$
- Conditional Independence
  - $P(X = x_i | Y = y_j) \neq P(X = x_i)$
  - $P(X = x_i | Y = y_j, Z = z_k) = P(X = x_i | Z = z_k)$
- Conditional Dependence
  - $P(X = x_i | Y = y_j) = P(X = x_i)$
  - $P(X = x_i | Y = y_j, Z = z_k) \neq P(X = x_i | Z = z_k)$

Joint Probability

$P(X = x_i, Y = y_j)$
"What is my belief that $X = x_i$ **and that** $Y = y_j$"

Joint Probability

Course code: **GJLOWD**

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice and participated in anti-nuclear demonstrations.

Which is more probable?

A. Linda is a bank teller.

B. Linda is a bank teller and is active in the feminist movement.

Tversky & Kahneman(1983)

Joint Probability

Course code: **GJLOWD**

$P(X = x_i, Y = y_j)$

"What is my belief that $X = x_i$ **and that** $Y = y_j$"

Which of the following is always true?

A. $P(X = x_i \text{ or } Y = y_j) \leq P(X = x_i, Y = y_j)$

B. $P(X = x_i \text{ or } Y = y_j) \geq P(X = x_i, Y = y_j)$

C. $P(X = x_i \text{ or } Y = y_j) = P(X = x_i, Y = y_j)$

D. *None of the above.*

Joint Probability

$P(X = x_i, Y = y_j)$

"What is my belief that $X = x_i$ **and that** $Y = y_j$"

$T \in \{t_1 = hot, t_2 = cool\}$

$W \in \{w_1 = sunny, w_2 = cloudy\}$

Joint Probability Table

	T = hot	T = cool
W = sunny	0.5	0.3
W = cloudy	0.05	0.15

Marginalization

$$P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$$

$$\sum_j P(W = w_i, T = t_j)$$

$T \in \{t_1 = hot, t_2 = cool\}$

W $\in \{w_1 = sunny, w_2 = cloudy\}$

Joint Probability Table

	T = hot	T = cool
W = sunny	0.5	0.3
W = cloudy	0.05	0.15

What is P(W=...)?

W = sunny	
W = cloudy	

Product Rule

Connects joint probability with conditional probability

$$P(X = x_i, Y = y_j) = P(X = x_i)\, P(Y = y_j | X = x_i)$$

$$P(X = x_i, Y = y_j) = P(Y = y_j)\, P(X = x_i | Y = y_j)$$

Product Rule Generalization

$$P\left(X = x_i, Y = y_j, Z = z_k, \ldots\right)$$

$$= P(X = x_i)P\left(Y = y_j | X = x_i\right)P\left(Z = z_k | X = x_i, Y = y_j\right) \ldots$$

This decomposition often makes reasoning easier.

Shorthand

Implied Universality:

$$P(X,Y) = P(X|Y)P(Y) = P(Y|X)P(X)$$

Implied Assignment:

$$P(x,y,z) = P(X = x, Y = y, Z = z)$$

Bayes' Rule

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Derive Bayes' Rule from the Product Rule.

$$P(x,y) = P(x)\,P(y|x)$$

$$P(y)\,P(x|y) = P(x)\,P(y|x)$$

$$\frac{P(T=1|C=1) \cdot P(C=1)}{P(T=1)}$$

Bayes' Rule example

- Let's assume breast cancer affects 1% of all patients $P(C=1) = 0.01$
- When a patient has cancer, a mammogram test comes back positive 90% of the time (true positive) $P(T=1|C=1)$
- When a patient does not have cancer, a mammogram test comes back positive 10% of the time (false positive) $P(T=1|C=0)$

A patient has a positive mammogram test. What is the probability the patient has cancer? $P(C=1|T=1)$

Define the following random variables:

T -> Test is positive $T=1$ $T=0$

C -> Patient has cancer $C=1$ $C=0$

Bayes' Rule example

- Let's assume breast cancer affects 1% of all patients -> $P(C=1) = 0.01$
- When a patient has cancer, a mammogram test comes back positive 90% of the time (true positive) -> $P(T=1|C=1) = 0.9$
- When a patient does not have cancer, a mammogram test comes back positive 10% of the time (false positive) -> $P(T=1|C=0) = 0.1$

A patient has a positive mammogram test. What is the probability the patient has cancer?

Define the following random variables:

T -> Test is positive

C -> Patient has cancer

Given: P(C=1), P(T=1|C=1), P(T=1|C=0)

What is $P(C = 1|T = 1)$?

$$P(C = 1|T = 1) = \frac{P(T=1|C=1)P(C=1)}{P(T=1)}$$ Bayes' Rule

What is the $P(T = 1)$?

$$P(T = 1) = \sum_c P(T = 1, C = c)$$ Marginalization

$$P(T = 1) = \sum_c P(T = 1|C = c)P(C = c)$$ Product Rule

$$P(T = 1) = P(T = 1|C = 1)P(C = 1) + P(T = 1, C = 0)P(C = 0)$$

What is the $P(C = 0)$?

$$P(C = 0) = 1 - P(C = 1)$$ Axiom

Conditioning on Background Evidence

- Product rule without background evidence: $P(X, Y) = P(Y|X)P(X)$
- Claim: $P(X, Y|E) = P(Y|X, E)P(X|E)$

Prove the conditional version of the product rule. (HW)

Conditioning on Background Evidence

- Product rule (conditional): $P(X, Y|E) = P(Y|X, E)P(X|E)$
- Bayes' rule (conditional):

$$P(X|Y, E) = \frac{P(Y|X, E)P(X|E)}{P(Y|E)}$$

- Marginalization (conditional):

$$P(X|E) = \sum_{y} P(X, Y = y|E)$$

That’s all folks!